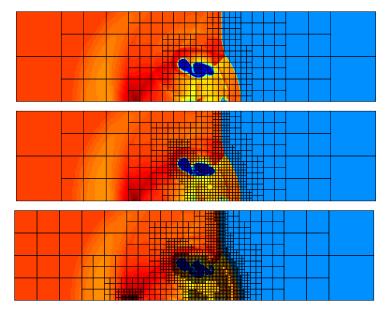
Active learning for cost-aware model reduction

Dmitry Duplyakin (U Utah and CU), **Jed Brown** (CU Boulder), Donna Calhoun (Boise State) jed.brown@colorado.edu

Copper Mountain Conference on Iterative Methods https://jedbrown.org/files/20180327-ActiveLearning.pdf

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AMR shock-bubble with 4, 5, and 6 levels



What goes wrong?

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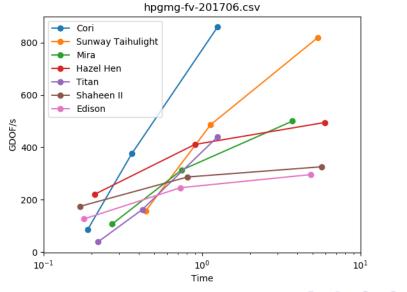
- Resubmit batch job, this time using more nodes.
- Tweak a refinement parameter.

On the last time step?

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- Checkpoint more often?
- Need to wait through the queue again.

New computer



Modeling

(response) =
$$f(x) + \mathcal{N}(0, \sigma_n^2)$$

- x user-relevant parameters
 - Physics: bubble size, density, shock intensity
 - Numerics: box size, max levels, refinement criteria

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- Machine: # nodes, MPI/OpenMP, compilers
- f Response
 - CPU time
 - Wall-clock time
 - Peak memory usage
 - ► Physics: ∆ entropy, decay time

 σ_n unbiased Gaussian noise

LOL Gaussian!

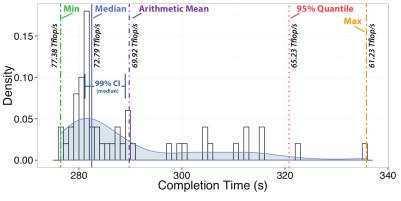


Figure 1: Distribution of 50 HPL measurement results.

[Hoefler & Belli, SC15]

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Gaussian process regression

$$p(f(x_{\star}) \mid X, y) \sim \mathcal{N}(\mu_{\star}, \sigma_{\star}^2)$$

$$\mu_{\star} = k_{\star}^{\mathsf{T}} K_{y}^{-1} y$$

$$\sigma_{\star}^{2} = k_{\star\star} - k_{\star}^{\mathsf{T}} K_{y}^{-1} k_{\star}$$

$$K_{y} = K + \sigma_{n}^{2} I$$

where

$$[K]_{ij} = k(x_i, x_j) = \sigma_f^2 \exp\left(-\frac{|x_i - x_j|^2}{2\ell^2}\right)$$

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in terms of *hyperparameters* ℓ , σ_f , σ_n .

Optimizing hyperparameters

The evidence provided by (X, y) in support of $(\ell, \sigma_t, \sigma_n)$ is quantified by

$$\mathscr{L}(\ell,\sigma_f,\sigma_n) = \log p(y \mid X, I, \sigma_f^2, \sigma_n^2) = -\frac{1}{2} \left(y^{\mathsf{T}} K_y^{-1} y + \log |K_y| \right) + C$$

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- Non-convex optimization problem
- Determinant $|K_{v}|$ due to normalization

Offline Active Learning

- Precompute database of features and responses
- Partition data (X, y) into Initial, Active, Test
- Compare many "trajectories" using different partitions

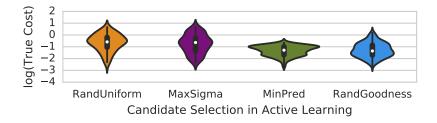
Algorithm

- Train GPR for each feature (e.g., cost and memory) in Initial set
- Repeat
 - 1. Consult GPR models to select next observation from Active

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- 2. Make that observation (incurring cost, etc.)
- 3. Retrain GPRs with new observation (including failure)

Selection procedure



RandUniform Uniform random sampling

MaxSigma Choose candidate with largest uncertainty

MinPred Maximize $\sigma_{\rm cost}/\mu_{\rm cost}$

RandGoodness Probability density $\sim \sigma_{
m cost}/\mu_{
m cost}$

RandGoodness with Memory Awarness As above, but exclude cases that violate memory bound

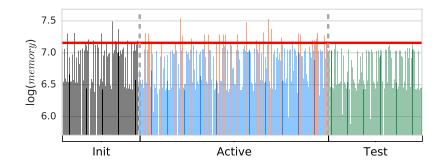
Metrics

Accuracy $RMSE = \sqrt{\frac{1}{n}} e^{T} e^{T}$

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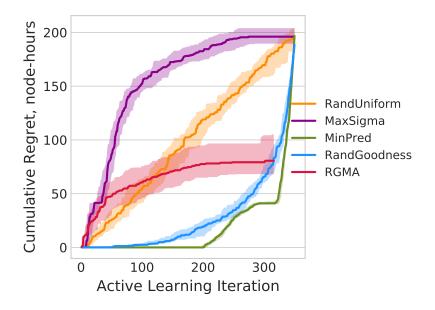
Cumulative Regret Costs incurred attempting failures

Memory limits



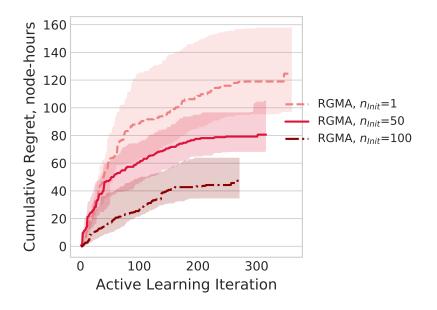
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Cumulative Regret by Algorithm



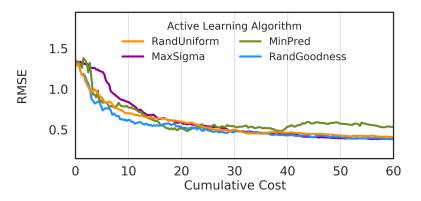
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Cumulative Regret by nInit



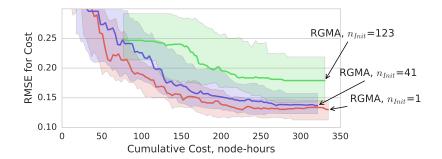
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RMSE vs Cumulative Cost



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RMSE vs Cumulative Cost



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Outlook

- Better measure of "risk" of exceeding memory bound
- Domain-specific kernel functions
- Non-Gaussian distributions
- Online mode
- Thanks to NERSC Edison and Cori, and to DOE ASCR

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